Exploration 7

Jaime Maxey

Make GSP constructions and script tools for construction of the tangent circles. Investigate, discuss, and state conjectures about the loci of the centers of the tangent circles.

We will first construct tangent circles. In order to do this I will first create two circles, with one inside the other. I will then draw a line through the center of the larger circle. I will also draw a line representing the radius of the smaller circle. At one of the ends of the larger circle's diameter, I will draw a circle that has the end point of the diameter at its center and a radius equal to the radius of the smaller circle.
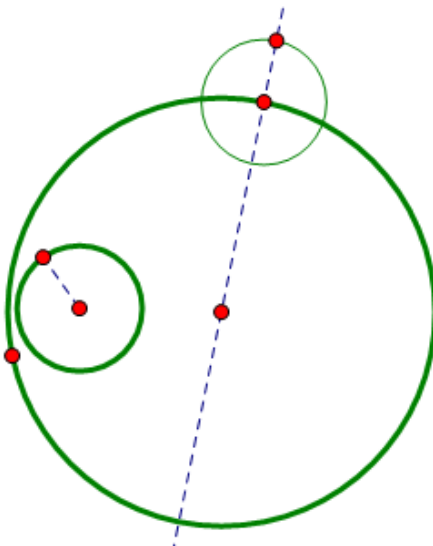

Next, I will construct a line segment from the center of the smaller circle inside the larger circle to the furthest point on the diameter of the third circle constructed. I will then find the midpoint of this line segment and draw a perpendicular line to the segment through the midpoint.
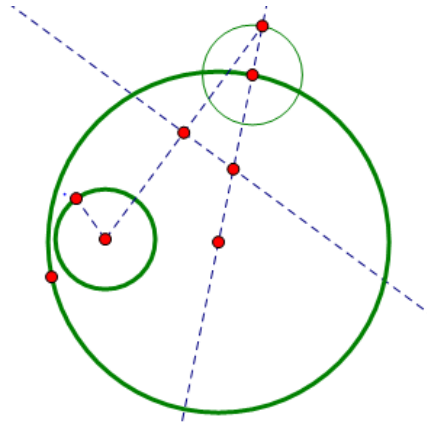
I will then mark the point where the diameter of the circle intersects with the perpendicular line we just created. This point will mark the center of the fourth circle that will be tangent to our large circle and our small circle inside the circle. Essentially we have extended the radius of the tangent circle inside the large circle by the radius of the small circle. In addition, the perpendicular bisector of the line segment from the small circle to the outside circle contains all of the points that are of equal distance to both the small and large circle. We can also notice that the segment that connects the center of the small circle to the point on the on the outside of the other small circle creates the base of an isosceles triangle, with the third point of the triangle being the center of the large circle.
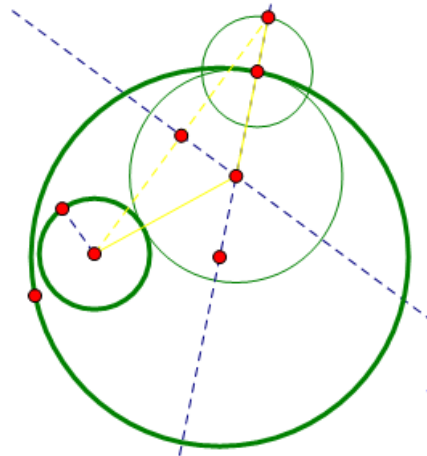
We will now investigate the locus of all possible solutions when the small circle is inside the large circle. We can visit this exploration in GSP in our second GSP file. Tracing the center of the tangent circle creates an ellipse of possible solutions as shown below.
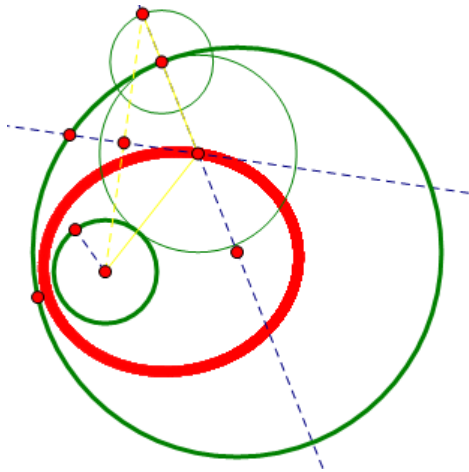
We will now investigate the locus of all possible solutions when the small circle is outside of the large circle. We can visit this exploration in GSP in our third GSP file. Tracing the center of the tangent circle creates a hyperbola of possible solutions as shown below.
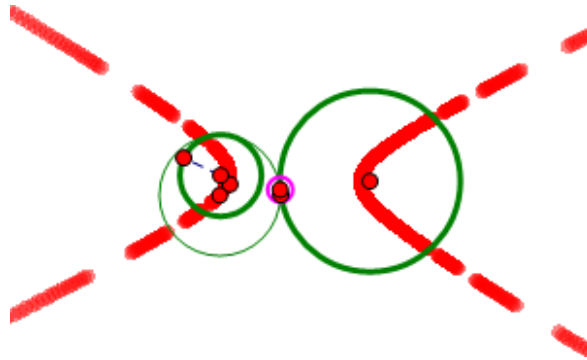
Last, we will investigate the locus of all possible solutions when the two circles overlap. We can visit this exploration in GSP in our fourth GSP file. Tracing the center of the tangent circle in this situation creates an ellipse of possible solutions.